Concept Analyst

VERIFICATION EXAMPLES

Version 3.0 beta

Concept Analyst Ltd.
Durham University
Department of Engineering
South Road
Durham DH1 3LE

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Contents

DISCLAIMER

This software is provided as a tool for an engineer. While it has undergone a series of tests, it is important to recognise that it is performing numerical simulations of physical phenomena, and that these involve approximations. An engineer performing these simulations with Concept Analyst does so at his/her own risk. Neither Concept Analyst, Ltd. nor the University of Durham shall be held responsible for the results obtained, nor for any consequential loss. Confirming the accuracy and/or usefulness of all the solutions is the responsibility of the licensee or user.

1

Introduction

This document presents some example problems that engineers may use as a basis to verify the results of the Concept Analyst software. For each example, the following shall be presented:

- A description of the geometry and loading
- Result(s) taken from another source
- Concept Analyst results

Users are encouraged to test their own installation of Concept Analyst using these and/or other examples in order to gain confidence in the results and to gain some intuition into the accuracy that is to be expected from different types of problem with different types of mathematical model.

Concept Analyst has been tested extensively. In each example there are usually several parameter combinations available (such as length, radii etc). In each case, these parameters are varied, and scaled versions of each model are subsequently tested, providing several hundred models for each example. For each example just a small sample of results is presented for verification.

2

Plate with hole

This is a classical stress concentration problem. A rectangular plate containing a central hole is subjected to uniaxial stress in the horizontal direction (Figure 2.1).

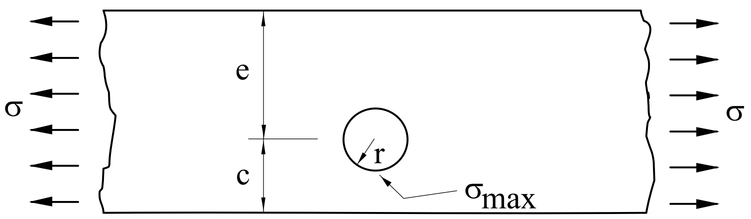

Figure 2.1. Plate with hole

The stress concentration gives rise to a peak stress at the bottom of the circular hole, as shown. The solutions of Peterson[1] are used to form a comparison.

Multiple combinations of these parameters defined in the figure (as well as scaled versions of each model) have been used for verification. Four sample cases are presented in this section. For the Concept Analyst model of this geometry, it is necessary to make some assumption about the length of the plate, since it is clearly impossible to sketch

[1] R.E. Peterson, Stress Concentration Factors, Wiley, 1974.

and analyse an infinite strip using the facilities in the program. Therefore the plate length, L, is an additional parameter that has been considered. For the results presented, a plate of 12 mm width is used.

A Concept Analyst model for this case is shown in Figure 2.2.

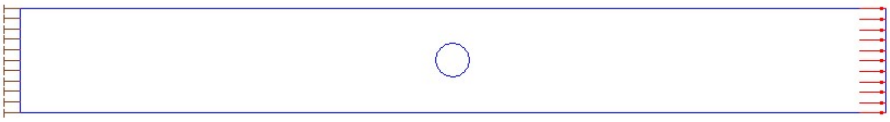

Figure 2.2. Concept Analyst model

Notice that no constraint is applied in the vertical direction. The program will apply a soft spring constraint (see the Concept Analyst User Guide) and this generally provides the most accurate results for problems exhibiting incomplete constraint. In other words, any constraint applied in the vertical direction would be changing the conditions under which the plate is loaded, and would therefore tend to invalidate the comparison.

Sample results are presented in Table 2.1 for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1.

L/mm	r/c	e/c	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
50	0.1	1	3.04	3.036	3.037	3.025	3.025
100	0.5	1	4.30	4.239	4.249	4.292	4.287
50	0.3	2	3.29	3.320	3.297	3.293	3.320
100	0.5	2	4.14	4.193	4.232	4.100	4.109

Table 2.1. Selected Concept Analyst Results for plate of width 12mm

Notes:

1. Some of the difference between Peterson's results and those of Concept Analyst is due to the fact that the infinite plate has been approximated by one of finite length. The comparison can be observed to be closer if a longer plate is used.

2. The Peterson results in this example are read from a graph and can be interpreted only within a coarse resolution.

3. The Concept Analyst results presented are those taken from the maximum level on an *x-y* boundary graph plot of maximum principal stress.

The examples given in Table 2.1 have been used to verify the option to specify boundary conditions as non-zero displacements. By taking displacements found on the right-hand side edge of the model and applying them as non-zero displacement boundary conditions, the resulting stress values along the right-hand edge of the new model should equal the load applied in the original model.

An x-y plot of the x-displacement of the right-hand side of the initial model (Figure 2.2) gives the displacement values for a new model (Figure 2.3), in which the traction boundary condition as been replaced with a non-zero displacement boundary condition.

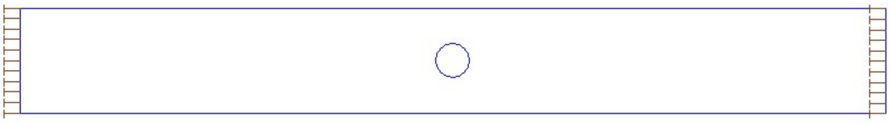

Figure 2.3. Concept Analyst model

Let *u* denote the *x*-displacement of the right-hand side nodes of the original model using the material properties for mild steel. *u* varies over the length of the line, and a mean value is applied now as a boundary condition in place of the traction that was previously applied to generate the results in Table 2.1. Table 2.2 shows the peak value of maximum principal stress on the hole perimeter using non-zero displacement boundary conditions. Coarse, standard and fine mesh settings were used. This peak stress is compared with the value predicted by Peterson for the original set of boundary conditions (an applied load) as for Table 2.1.

L/mm	r/c	e/c	σ_1 max Peterson	*u*/mm Coarse	σ_1 max Coarse	*u*/mm Standard	σ_1 max Standard	*u*/mm Fine	σ_1 max Fine
50	0.1	1	304	0.02429	302.5	0.02429	303.7	0.02429	303.6
100	0.5	1	430	0.05294	429.2	0.05291	425.0	0.05292	423.9
50	0.3	2	329	0.024745	325.7	0.02474	326.1	0.024745	328.3
100	0.5	2	414	0.05018	402.0	0.05020	414.3	0.050205	411.1

Table 2.2 Concept Analyst Results for non-zero displacement boundary condition problems (Standard Mesh). Stresses in MPa.

3

Plate with uneven holes

This example is also a comparison with results from Peterson. It involves the stress concentrations around two holes of different diameter in an infinite plate under a uniaxial stress field (Figure 3.1).

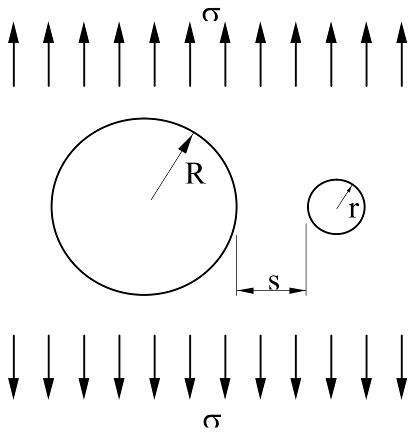

Figure 3.1. Infinite plate with two uneven holes

For the Concept Analyst model of this geometry, it is necessary to make some assumption about the size of the plate, since it is clearly impossible to sketch and analyse an infinite plate using the facilities in the program.

A Concept Analyst model for this case is shown in Figure 3.2.

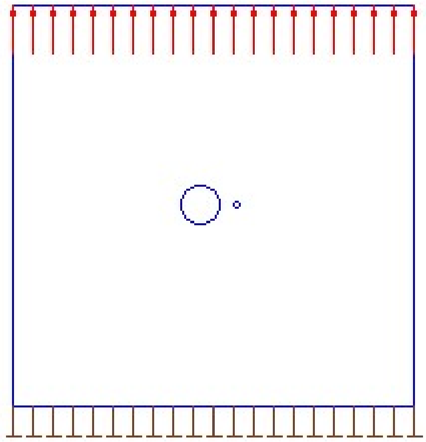

Figure 3.2. Concept Analyst model

Notice that no constraint is applied in the horizontal direction. The program will apply a soft spring constraint (see the Concept Analyst User Guide) and this generally provides the most accurate results for problems exhibiting incomplete constraint. In other words, any constraint applied in the horizontal direction would be changing the conditions under which the plate is loaded, and would therefore tend to invalidate the comparison.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (Table 3.1). In these examples, an aluminium plate of dimensions 10 x 10 mm contains two holes of radius R and r.

R/r	R/mm	s/r	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
1	0.1	3	3.1	3.039	3.037	3.034	3.034
1	0.4	3	3.1	3.097	3.096	3.095	3.095
5	0.1	4	3.5	3.485	3.485	3.485	3.487
5	0.4	4	3.5	3.517	3.518	3.518	3.520
10	0.1	5	4.1	4.110	4.110	4.110	4.114
10	0.4	5	4.1	4.145	4.149	4.149	4.153

Table 3.1. Selected Concept Analyst results for plate dimensions 10mm x 10mm where the centre of the plate coincides with the mid-point of dimension ‘s’ in Fig 3-1

Notes:

1. Some of the difference between Peterson's results and those of Concept Analyst are due to the fact that the infinite plate has been approximated by one of finite length. The comparison can be observed to be closer if a longer plate is used.

2. The Peterson results in this example are read from a graph and can be interpreted only within a coarse resolution.

3. If the centre of the plate is not coincident with the mid-point of dimension 's' in Figure 3.1, the results in Table 3.1 will be slightly different. This will be particularly the case in the models for which R = 0.4mm, i.e. the hole is no longer very small in comparison with the plate.

4

Rectangular hole with rounded corners

This is the third example to compare the results of Peterson with those of Concept Analyst. It involves the stress concentrations around a rectangular hole, with rounded corners, in an infinite plate under a biaxial tensile stress field (Figure 4.1).

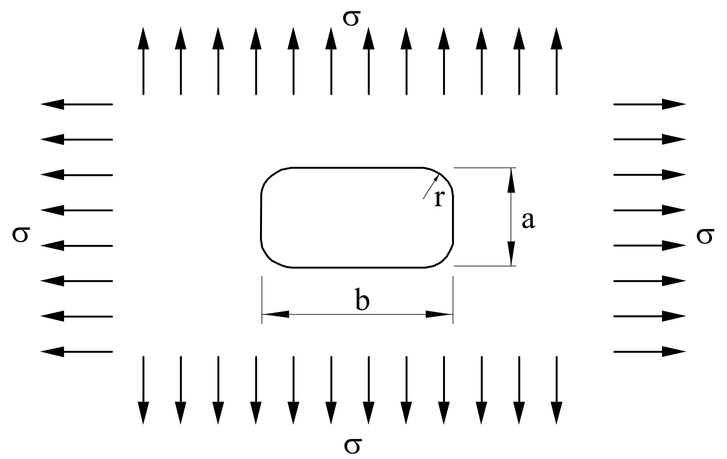

Figure 4.1. Rectangular hole with rounded corners

For the Concept Analyst model of this geometry, it is necessary to make some assumption about the size of the plate, since it is clearly impossible to sketch and analyse an infinite plate using the facilities in the program.

A Concept Analyst model for this case is shown in Figure 4.2.

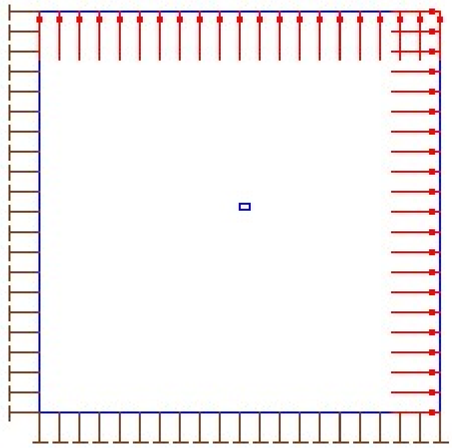

Figure 4.2. Concept Analyst model

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (Table 4.1). In these examples, a plate of dimensions 10 x 10 mm contains a hole of varying b- and r-dimensions, where dimension a = 0.1 mm.

b/a	r/b	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
1.0	0.10	4.87	4.829	4.860	4.860	4.859
1.5	0.12	4.11	4.072	4.081	4.081	4.079
2.0	0.17	3.23	3.302	3.313	3.313	3.302
2.5	0.10	4.21	4.192	4.193	4.193	4.191
3.0	0.14	3.68	3.631	3.649	3.649	3.630

Table 4.1. Selected Concept Analyst results for plate dimensions 10mm x 10mm

Notes:

1. Some of the difference between Peterson's results and those of Concept Analyst are due to the fact that the infinite plate has been approximated by one of finite length. The comparison can be observed to be closer if a longer plate is used.

2. The Peterson results in this example are read from a graph and can be interpreted only within a coarse resolution.

5

Block under self-weight

This example presents a comparison with the theoretical deflection of a 10m × 7m rectangular block of material under the action of gravitational load. The situation is illustrated in Figure 5.1. We consider the block to have unit thickness.

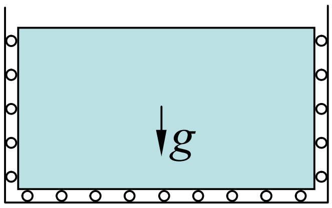

Figure 5.1 Block under self-weight

In this section, Concept Analyst is used to model the block to enable simple determination of the maximum deflection and stress distribution.

The Concept Analyst model is shown in Figure 5.2.

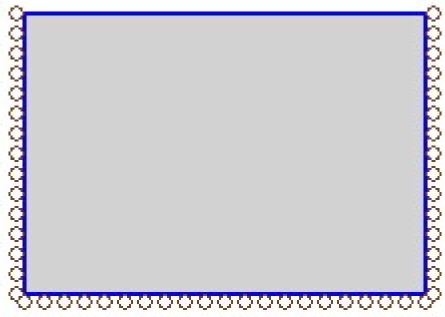

Figure 5.2 Concept Analyst model

The self-weight is activated using the *Loading - Self Weight* command. We set the *y*-direction acceleration to be $g = -9.81\text{m/s}^2$ (negative because it acts downwards and *y* is positive upwards). Note that the self-weight function can also be used to provide acceleration in any chosen direction.

The theoretical deflection of the block is not as straightforward as it looks because of the constraint against horizontal expansion. The deflection at the top of the block is given by

$$v = \frac{(1-\nu^2)\rho g H^2}{2E}$$

where H is the vertical height of the block, E is Young's Modulus and ρ is the mass density of the material.

The vertical stress at a height y from the base of the block is given by

$$\sigma_y = (H - y)\rho g$$

In the following examples, a block is varied in height. Results are given in Tables 5.1 and 5.2 for the displacement, v, at the top of the block and the vertical stress, σ_y, at the base. We use different materials and different unit sets and use the standard mesh setting throughout, without adaptivity.

Unit set	Material	H	v	
			Theory	C.A.
N, m, Pa	Mild steel	7 m	-8.286×10^{-6} m	-8.294×10^{-6} m
N, mm, MPa	Mild steel	7000 mm	-8.286×10^{-3} mm	-8.294×10^{-3} mm
N, m, Pa	Aluminium	7 m	-8.291×10^{-6} m	-8.291×10^{-6} m
N, mm, MPa	Aluminium	7000 mm	-8.291×10^{-3} mm	-8.291×10^{-3} mm

Table 5.1. Selected Concept Analyst results (displacement v)

Unit set	Material	H	σ_y	
			Theory	C.A.
N, m, Pa	Mild steel	7 m	-539100 Pa	-539000 Pa
N, mm, MPa	Mild steel	7000 mm	-0.5391 MPa	-0.5390 MPa
N, m, Pa	Aluminium	7 m	-186100 Pa	-186100 Pa
N, mm, MPa	Aluminium	7000 mm	-0.1861 MPa	-0.1861 MPa

Table 5.2. Selected Concept Analyst results (stress σ_y)

6

Thick-walled cylinder

This example compares the Concept Analyst results with the theoretical equations of elasticity in internally pressurised thick-walled cylinders. One quarter of the cylinder is to be considered (Figure 6.1).

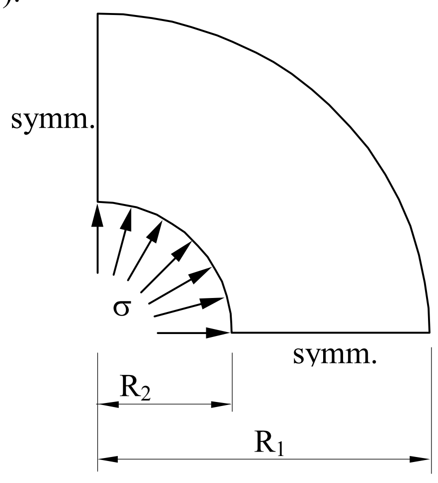

Figure 6.1. Thick walled cylinder

Concept Analyst is used to model a symmetrical section with an internal pressure of 100 MPa (Figure 6.2).

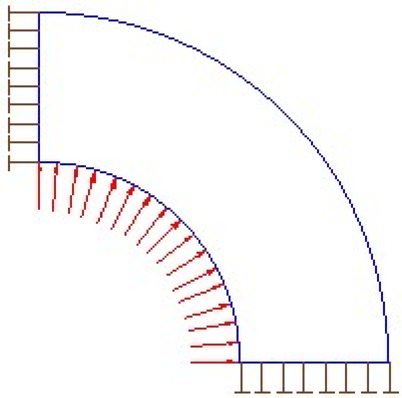

Figure 6.2. Concept Analyst model

Thick cylinder theory states that at any radial coordinate, r, the radial and hoop stress components, σ_θ and σ_r respectively, are given by the Lamé equations

$$\sigma_\theta = A + \frac{B}{r^2} \qquad \sigma_r = A - \frac{B}{r^2}$$

where A and B are constants for any cylinder/pressure, and these can be determined from the particular boundary conditions. In this case, the boundary conditions are zero radial stress at the outer radius r = R_1 and a radial stress equal to the internal pressure 100 MPa at the inner radius r = R_2. These give constants A and B. The maximum stress in the cylinder is the hoop stress at the inner radius, and this may be readily determined from the above equation by substituting values of r and determining constants A and B in each case.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (Table 6.1).

R_1	R_2	K_t Theory	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
100	50	1.667	1.667	1.667	1.668	1.667
200	50	1.133	1.134	1.134	1.138	1.135
300	100	1.250	1.250	1.251	1.252	1.253
475	100	1.093	1.093	1.094	1.098	1.094
788	175	1.104	1.104	1.105	1.108	1.105

Table 6.1. Selected Concept Analyst results

This shows very good correlation with theory.

7

Plate with opposite notches

This example is also a comparison with results from Peterson. It involves the stress concentrations around two notches of opposite identical radii in an infinite plate under a uniaxial stress field (Figure 7.1).

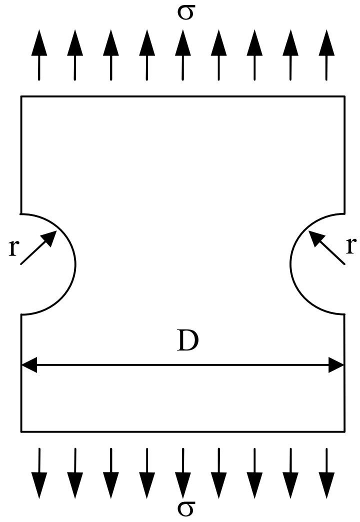

Figure 7.1. Plate with opposite semi-circular notches

For the Concept Analyst model of this geometry, it is necessary to make some assumption about the size of the plate, since it is clearly impossible to sketch and analyse an infinite plate using the facilities in the program.

A Concept Analyst model for this case is shown in Figure 7.2.

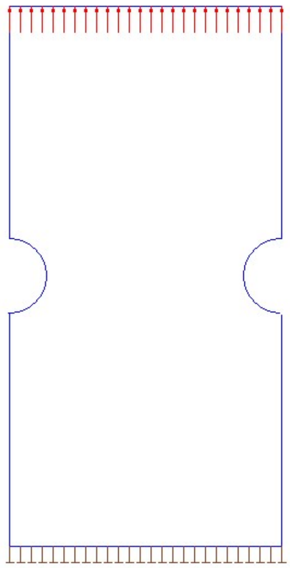

Figure 7.2. Concept Analyst model

Notice that no constraint is applied in the horizontal direction. The program will apply a soft spring constraint (see the Concept Analyst User Guide) and this generally provides the most accurate results for problems exhibiting incomplete constraint. In other words, any constraint applied in the horizontal direction would be changing the conditions under which the plate is loaded, and would therefore tend to invalidate the comparison.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (Table 7.1). In these examples, a plate of 100 mm length and varying widths contains two semi-circular notches with a range of radii.

D / mm	r / mm	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
20	7	4.20	4.213	4.232	4.232	4.232
30	6	3.10	3.107	3.103	3.103	3.103
40	7	3.05	3.070	3.066	3.066	3.066
40	12	3.55	3.548	3.551	3.551	3.551
50	10	3.28	3.108	3.104	3.104	3.104

Table 7.1. Selected Concept Analyst results for plate of length 100mm

8

Plate with adjacent notches

This example is also a comparison with results from Peterson. It involves the stress concentrations around two adjacent notches of identical radii in an infinite plate under a uniaxial stress field (Figure 8.1).

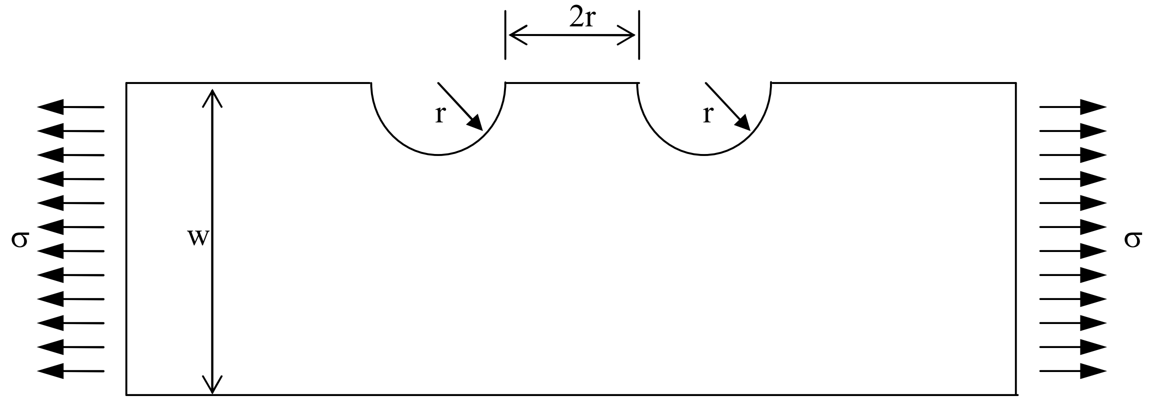

Figure 8.1. Plate with adjacent semi-circular notches

For the Concept Analyst model of this geometry, it is necessary to make some assumption about the size of the plate, since it is clearly impossible to sketch and analyse an infinite plate using the facilities in the program.

A Concept Analyst model for this case is shown in Figure 8.2.

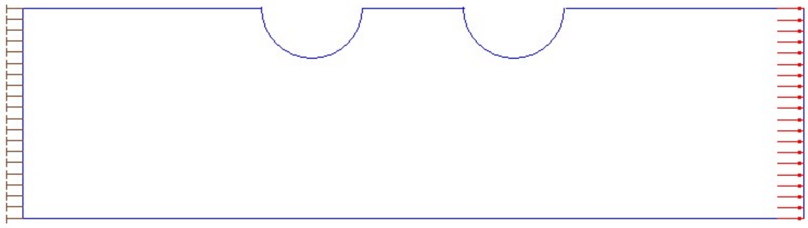

Figure 8.2. Concept Analyst model

Notice that no constraint is applied in the vertical direction. The program will apply a soft spring constraint (see the Concept Analyst User Guide) and this generally provides the most accurate results for problems exhibiting incomplete constraint. In other words, any constraint applied in the horizontal direction would be changing the conditions under which the plate is loaded, and would therefore tend to invalidate the comparison.

Sample results are presented for fine, standard and coarse mesh density settings, in the form of stress concentration factors, K_t, calculated from the maximum principal stress, σ_1 (Table 8.1). In these examples, a plate of 100 mm length and varying widths contains two semi-circular notches with a range of radii.

w / mm	r / mm	w / r	K_t Peterson	K_t Fine Mesh	K_t Standard Mesh	K_t Coarse Mesh	K_t Coarse Adaptive
3	0.27	11.1	2.90	2.969	2.980	2.972	2.969
6	0.90	6.67	3.43	3.433	3.427	3.427	3.433
12	1.44	8.33	3.20	3.225	3.216	3.174	3.178
18	1.08	16.7	2.86	2.893	2.889	2.889	2.893

Table 8.1. Selected Concept Analyst results of strip length 100mm

Notes:

1. Some of the differences between Peterson's results and those of Concept Analyst are due to the fact that the infinite plate has been approximated by one of finite length. The comparison can be observed to be closer if a longer plate is used.

2. The Peterson results in this example are read from a graph and can be interpreted only within a coarse resolution.

9

Plate with edge crack

This example is a simple edge crack in a square plate of side 100 mm, under uniaxial uniform tensile stress σ = 100 MPa, illustrated in figure 9.1. Comparison will be made with stress intensity factor results read from a graph in *Compendium of Stress Intensity Factors*, by Rooke & Cartwright.

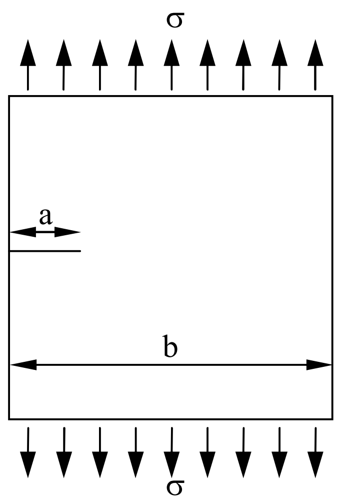

Figure 9.1. Square plate with edge crack

A Concept Analyst model for this case is shown in Figure 9.2.

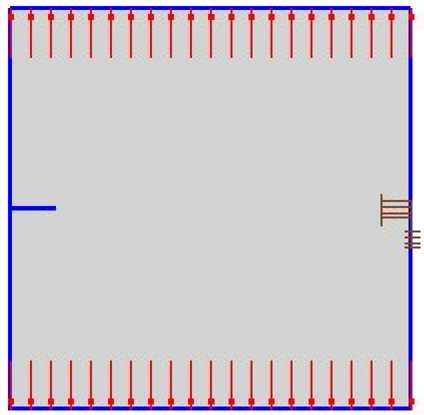

Figure 9.2. Concept Analyst model

Notice that the displacement constraints in both *x* and *y*-directions are applied over a short line segment in the middle of the right hand side of the plate. The coordinates of the ends of this line segment in the verification study are (100, 48) and (100, 52), the plate being bounded by $0 < (x,y) < 100$ mm.

Sample results are presented in Table 9.1 for fine, standard and coarse mesh density settings, in the form of stress intensity factor, K_I. Comparison is made with stress intensity factors calculated by reading from a curve in Rooke & Cartwright.

a / mm	a/b	K_I (Rooke) MPa√m	K_I Fine Mesh	K_I Standard Mesh	K_I Coarse Mesh
10	0.1	22.5	21.9	21.9	21.9
20	0.2	37.1	37.5	37.3	37.5
30	0.3	56.2	57.0	57.1	57.3
40	0.4	81.9	82.7	82.8	83.2

Table 9.1 Selected Concept Analyst results

Notes:

1. The value of K_I/K_0 should be observed to converge to 1.12 as a→0. This can be demonstrated in Concept Analyst using short crack lengths a.

a (mm)	K_I/K_0
0.5	1.120
1.0	1.125

2. The Rooke & Cartwright results in this example are read from a graph and can be interpreted only within a coarse resolution.

10

Pin/hole contact

Concept Analyst has the capability to solve the non-linear problem of a loaded circular pin in a circular hole. In this example, we consider such an arrangement in an infinite plate. A circular hole of radius 1 contains a pin with perfect fit so that the pin radius is also 1. The contact is frictionless, and a load of 1 applied in the x-direction to the pin.

A Concept Analyst model is shown in Figure 10.1. In order to consider an infinite plate, the plate is given a size of 100x100. Constraints against displacement in both x and y directions are applied on all four edges.

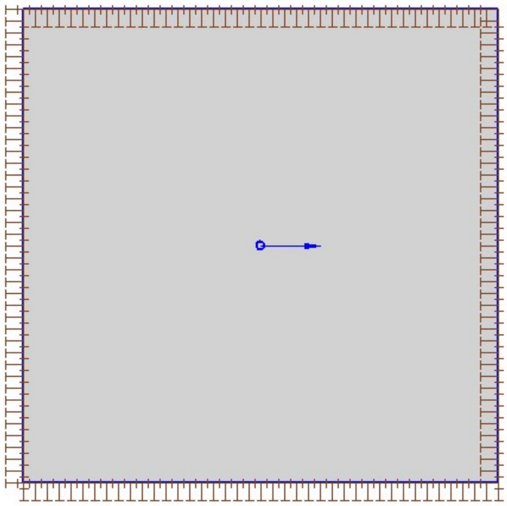

Figure 10.1 Plate with pin load of 1 applied in a circular hole.

The results are the same for coarse, standard and fine meshes because for pin/hole contact problems the same mesh is applied to ensure good resolution of the contact problem.

We compare in Table 10.1 the maximum contact stress between the pin and hole against the analytical solution from B. Noble and M.A. Hussain (1969), Exact solution of certain dual series for indentation and inclusion problems, International Journal of Engineering Science, 7:1149-1161.

	Maximum Contact Stress
Analytical solution	-0.5997
Concept Analyst	-0.5974

Table 10.1. Results comparison for non-linear pin/hole contact.

11

Limitations

It should always be recognised that the results produced by Concept Analyst are an approximation to physical behaviour.

Like many forms of numerical approximation, the accuracy of the solution is directly related to the quality of the approximation, and this is in turn related to the time and effort invested in the approximation.

For programs like Concept Analyst, this essential trade-off is found in the definition of the ‘mesh’. Finite element and boundary element software systems all use elements (of one sort or another) to describe the geometry and results. It is usually the case that using more elements will give rise to better results, but will require more time and greater usage of computational resources than the coarser mesh.

Most analysis systems like this leave to the user this decision about the number of elements to use. This has the advantage that the user is free to undertake what is called a ‘convergence’ analysis, in which models of increasing complexity are run, and when the solutions do not change markedly from one run to the next, it may be assumed that convergence has been achieved and the numerical model is proving to be adequate.

On the other hand, leaving the decision to the user requires that user to be reasonably expert in the use of this technology. Concept Analyst takes the approach that these important decisions should be made automatically by the program according to some rules contained within the software algorithms (although a limited convergence analysis is still available through the use of the coarse, standard and fine mesh density settings).

This has the advantage of ease and speed of use, particularly in the hands of non-expert stress analysts. However, it is possible that a geometry might cause the program difficulties in automatically defining a suitable mesh of boundary elements for that particular problem